(d) Adding two spin-½ angular momenta: Revisited

$$s_1 = 1/2\ ,\ \underbrace{m_{s_1} = \pm 1/2}_{|\uparrow\rangle_1\ ,\ |\downarrow\rangle_1}\ ;\quad s_2 = 1/2\ ,\ \underbrace{m_{s_2} = \pm 1/2}_{|\uparrow\rangle_2\ ,\ |\downarrow\rangle_2}$$

- 4 possibilities: $|1/2, \underline{m_{s_1} = \pm 1/2}\ ;\ 1/2, \underline{m_{s_2} = \pm 1/2}\rangle$ OR $|m_{s_1} ; m_{s_2}\rangle$

	$\alpha(1)\alpha(2)$	$\beta(1)\beta(2)$	$\alpha(1)\beta(2)$	$\alpha(2)\beta(1)$
OR	$\lvert +1/2 ; +1/2\rangle$	$\lvert -1/2 ; -1/2\rangle$	$\lvert +1/2 ; -1/2\rangle$	$\lvert -1/2 ; +1/2\rangle$
OR	$\lvert\uparrow\rangle_1 \lvert\uparrow\rangle_2$	$\lvert\downarrow\rangle_1 \lvert\downarrow\rangle_2$	$\lvert\uparrow\rangle_1 \lvert\downarrow\rangle_2$	$\lvert\downarrow\rangle_1 \lvert\uparrow\rangle_2$
	symmetric	symmetric	Neither symmetric NOR anti-symmetric	

[OK! Can go with Anti-sym $\psi_{spatial}$]

[no good for constructing 2-electron wavefns]

- Invoke total spin AM $\Rightarrow$ $S=0$ and $S=1$

  $|S, m_s\rangle$ also labels 4 states

- Already know that: <u>Singlet</u> state $|S=0, m_s=0\rangle$ is

$$\psi_{spin}^{(S=0)} \text{ or } |S=0, m_s=0\rangle = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) - \alpha(2)\beta(1)\right]$$

$$= \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\right] \quad (40)$$

(<u>anti-symmetric</u>)

[vector model: two spins tend to anti-align]

$$\boxed{\text{Anti-symmetric } \psi_{spin} \text{ goes with symmetric } \psi_{spatial}}$$

- Triplet States: $\underbrace{|S=1, m_s=1\rangle}_{}$, $\underbrace{|S=1, m_s=-1\rangle}_{}$, $\underbrace{|S=1, m_s=0\rangle}_{}$

- Easy to see that: $\alpha(1)\alpha(2)$ or $|\uparrow\rangle_1 |\uparrow\rangle_2$ $\quad$ $\beta(1)\beta(2)$ or $|\downarrow\rangle_1 |\downarrow\rangle_2$ $\quad$ $\boxed{\text{What is this?}}$

($\because$ z-components add up to give $m_s=1$) $\quad$ ($\because$ z-components add up to give $m_s=-1$)

What is $|S=1, m_s=0\rangle$?

The only combination left is a superposition of $|\uparrow\rangle_1|\downarrow\rangle_2$ and $|\downarrow\rangle_1|\uparrow\rangle_2$

$$\psi_{spin}^{(S=1, m_s=0)} \text{ or } |S=1, m_s=0\rangle = \frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2) + \alpha(2)\beta(1)\right] \quad \text{(Symmetric)}$$

$$= \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2\right] \quad (43)$$

∴ The $\underline{S=1}$ (triplet) states are symmetric!

$\alpha(1)\,\alpha(2)$ $\left[|\uparrow\rangle_1 |\uparrow\rangle_2\right]$

$(S=1,\ \underline{m_s=1})$

$\beta(1)\,\beta(2)$ $\left[|\downarrow\rangle_1 |\downarrow\rangle_2\right]$

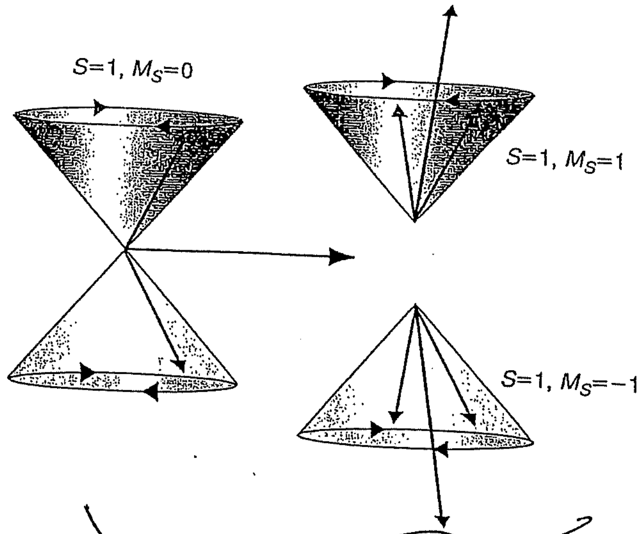
$(S=1,\ \underline{m_s=-1})$

$\frac{1}{\sqrt{2}}\left[\alpha(1)\beta(2)+\beta(1)\alpha(2)\right]$

OR $\frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2+|\downarrow\rangle_1|\uparrow\rangle_2\right]$

$(S=1,\ \underline{m_s=0})$

Symmetric ψ_{spin} goes with anti-symmetric $\psi_{spatial}$ (44)



triplet states

Vector Model
S=1 states
[tend to align]

Take-Home Message

$S=0,\ m_s=0^{+}$: $\frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\alpha(2)\beta(1)]$ Antisymmetric spin part "spin singlet"

$$S=1,\quad \left.\begin{array}{ll} m_s=1 & \alpha(1)\alpha(2) \\ m_s=0^{+} & \frac{1}{\sqrt{2}}[\alpha(1)\beta(2)+\alpha(2)\beta(1)] \\ m_s=-1 & \beta(1)\beta(2) \end{array}\right\} \text{Symmetric spin parts "spin triplet"} \qquad (45)$$

for adding two spin-half AM's

+ These states are interesting superposition of $|\uparrow\rangle_1|\downarrow\rangle_2$ & $|\downarrow\rangle_1|\uparrow\rangle_2$. They are quantum <u>entangled states</u>.

Summary of Sec. G

- N-electron wavefunction can be written as a Slater Determinant that gaurantees anti-symmetry (single-particle states are invoked)
- Pauli Exclusion Principle is a consequence [must be anti-symmetric]
- 2-electron wavefunction can be factorized $\psi_{2\text{-electron}} = \psi_{spatial} \cdot \psi_{spin}$
- 2-electron ψ_{spin} is related to adding to spin-½ AM's
- $S=0$ ($m_s=0$) (singlet) has antisymmetric ψ_{spin}
- $S=1$ ($m_s=1,0,-1$) (triplet) has symmetric ψ_{spin}